Module 16 Linear Algebra

Module title	Linear Algebra
Module NFQ level (only if an NFQ level can be demonstrated)	6
Module number/reference	BSCH-LA
Parent programme(s)	Bachelor of Science (Honours) in Computing Science
Stage of parent programme	Stage 2
Semester (semester1/semester2 if applicable)	Semester 2
Module credit units (FET/HET/ECTS)	ECTS
Module credit number of units	5
List the teaching and learning modes	Direct, Blended
Entry requirements (statement of knowledge, skill and competence)	Learners must have achieved programme entry requirements.
Pre-requisite module titles	BSCH-FC
Co-requisite module titles	None
Is this a capstone module? (Yes or No)	No
Specification of the qualifications (academic, pedagogical and professional/occupational) and experience required of staff (staff includes workplace personnel who are responsible for learners such as apprentices, trainees and learners in clinical placements)	Qualified to as least a Bachelor of Science (Honours) level in Computer Science or equivalent and with a Certificate in Training and Education (30 ECTS at level 9 on the NFQ) or equivalent.
Maximum number of learners per centre (or instance of the module)	60
Duration of the module	One Academic Semester, 12 weeks teaching
Average (over the duration of the module) of the contact hours per week	3
Module-specific physical resources and support required per centre (or instance of the module)	One class room with capacity for 60 learners along with one computer lab with capacity for 25 learners for each group of 25 learners

Analysis of required learning effort		
	Minimum ratio teacher / learner	**Hours**
Effort while in contact with staff		
Classroom and demonstrations	1:60	18
Monitoring and small-group teaching	1:25	18
Other (specify)		
Independent Learning		
Directed e-learning		
Independent Learning		34
Other hours (worksheets and assignments)		55
Work-based learning – learning effort		
Total Effort		125

Allocation of marks (within the module)					
	Continuous assessment	**Supervised project**	**Proctored practical examination**	**Proctored written examination**	**Total**
Percentage contribution	60%			40%	100%

Module aims and objectives

A key objective of this module is to give learners an in depth understanding of those areas of discrete mathematics that are relevant to the study of computing. It builds on the work covered as part of the first year foundations module.

Minimum intended module learning outcomes

On successful completion of this module, the learner will be able to:

1. Perform calculations using algebraic expressions
2. Define various mathematical structures and perform operations on them
3. Define different types of functions along with their properties
4. Perform calculations using modular arithmetic
5. Solve sets of linear equations using matrices
6. Perform calculations using vectors
7. Identify various types of graphs and describe their properties

Rationale for inclusion of the module in the programme and its contribution to the overall MIPLOs

Mathematics is a fundamental part of computing science. No matter what area of ICT a practitioner works in a solid understanding of mathematical concepts is essential. The work covered here provides a basis for the study of relational databases and formal specification (sets, relations and functions), graphics (vectors and matrices), telecommunications (matrices), and concurrent programming (matrices). Appendix 1 of the programme document maps MIPLOs to the modules through which they are delivered.

Information provided to learners about the module

Learners receive a programme handbook to include module descriptor, module learning outcomes (MIMLO), class plan, assignment briefs, assessment strategy and reading materials.

Module content, organisation and structure

Sets, Relations & Functions

- Sets: definition, typed set theory, equality, sub-set, intersection, union, difference, power set, properties of sets;
- Relations: definition, relation as set of tuples, domain, range, domain restriction and subtraction, range restriction and subtraction, inverse, types of relation: reflexive, symmetric and transitive, equivalence relations
- Functions: definition, domain, range, inverse, domain subtraction and restriction, function overriding, composition of functions, types of function: injective(one-to-one), surjective(onto), bijective

Vectors

- Scalar and vector quantities; equality of vectors; vector addition, unit vector, orthogonal basis, components of a vector in terms of unit vector, scalar product, vector product, angle between two vectors.

Matrices

- Matrix definition, order, types; equality; addition, subtraction, multiplication by a scalar, multiplication, transpose, inverse; solutions of sets of linear equations; transformation matrices.

Graph Theory

- Graph definition. Types of graphs: complete, directed, undirected, cyclic, acyclic, bipartite, planar etc. Graph representation in a computer: adjacency lists, adjacency matrices.

Module teaching and learning (including formative assessment) strategy

The module is delivered through a combination of lectures and tutorials. The learners work on worksheets throughout the module that build on the learning in lectures. The emphasis is on developing practical ability at mathematical reasoning based on sound theoretical knowledge.

The module assessment consists of three open book examinations (60%), and a closed book final examination (40%). Each open book examination has a value of 20%. These exams are held when a distinct identifiable piece of work has been completed on the module.

Timetabling, learner effort and credit

The module is timetabled as one 1.5-hour lecture per week and one 1.5-hour practical session per week.

Continuous assessment spreads the learner effort to focus on small steps and helps to ensure learner engagement over the course of the module.

There are 36 contact hours made up of 18 lectures and 18 practical sessions delivered over 12 weeks with both taking place in a classroom. The learner will need 34 hours of independent effort to further develop the skills and knowledge gained through the contact hours. An additional 55 hours are set aside for learners to work on class tests that must be completed for the module.

The team believes that 125 hours of learner effort are required by learners to achieve the MIMLOs and justify the award of 5 ECTS credits at this stage of the programme.

Work-based learning and practice-placement

There is no work based learning or practice placement involved in the module.

E-learning

The college VLE is used to disseminate notes, advice, and online resources to support the learners. The learners are also given access to Lynda.com as a resource for reference.

Module physical resource requirements

Requirements are for a classroom for 60 learners equipped with a projector.

Reading lists and other information resources

Recommended Text

Stroud, K. A. and Booth, D. J. (2013) *Engineering Mathematics*. Basingstoke: Palgrave MacMillan.

Secondary Reading:

Singh, K. (2013) *Linear Algebra: Step by Step*. Oxford: OUP.

Lipson, M., Lipschutz, S. (2012) *Schaum's Outline of Linear Algebra*. London: McGraw-Hill Education

Specifications for module staffing requirements

For each instance of the module, one lecturer qualified to at least Bachelor of Science (Honours) in Computer Science or equivalent, and with a Certificate in Training and Education (30 ECTS at level 9 on the NFQ) or equivalent.. Industry experience would be a benefit but is not a requirement.

Learners also benefit from the support of the programme director, programme administrator, learner representative and the Student Union and Counselling Service.

Module Assessment Strategy

The assignments constitute the overall grade achieved, and are based on each individual learner's work. The continuous assessments provide for ongoing feedback to the learner and relates to the module curriculum.

No.	Description	MIMLOs	Weighting
1	3 class tests which will assess a subset of the content for this module in a similar environment to the final exam. Class test 1 will assess learning outcomes 1, 2, 3 Class test 2 will assess learning outcomes 4, 5, 6 Class test 3 will assess learning outcomes 1, 2, 7	1- 7	20% each
2	Written exam that tests the theoretical aspects of the module	1- 7	40%

All repeat work is capped at 40%.

Sample assessment materials

Note: All assignment briefs are subject to change in order to maintain current content.

Class Test 1

Q1. Given the sets A={1,2,4,8,16}, B={2,3,6,9,15}, C={1,2,3,5,8,13,21}

Find: (i) $A \cup B$

(ii) $A \cap C$

(iii) $(A \cap B) \cup C$

(iv) $B \setminus A$

(v) $|A \cap B \cap C|$

(10 marks)

Q2. A survey was carried out with 60 people with respect to three car models: Ford, Toyota and Opel.

Represent the following information on a Venn diagram and calculate how many people liked all three car models?

16 like Ford, 25 like Toyota and 19 like Opel.

4 like none of the cars.

8 like Ford and Toyota, but not Opel.

4 like Toyota and Opel, but not Ford.

2 like Ford and Opel, but not Toyota.

(10 marks)

Q3.

(i) Solve for x: $\log_3(2x+3) + \log_3(x) = 2$.

(ii) Solve for x: $3^{(3x+3)} = 81$

(10 marks)

Q4. What does it mean for a function to be injective? Give an example of a function that is injective and an example of a function that is not injective.

(10 marks)

Q5. Graph the function f(x): $2x^2 - x + 9$ in the domain $-2 \leq x \leq 2$ with increments of 0.5.

(10 marks)

Class Test 2

Q1. Given the Matrices: $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $B = \begin{pmatrix} -1 & -2 \\ -3 & -4 \\ -5 & -6 \end{pmatrix}$ $C = \begin{pmatrix} 9 & 8 & 7 \\ -1 & -2 & -3 \end{pmatrix}$

(i) Calculate (A + B)

(ii) Calculate (2B – A)

(iii) Calculate (C)(A)

(10 marks)

Q2. Using Cramer's Rule, solve the following set of equations:

$$2x - 3y + z = 5$$
$$x + 2y + z = -1$$
$$x - 3y + 2z = 1$$

(10 marks)

Q3.

If $A = \begin{pmatrix} 3 & 5 & -2 \\ 6 & 10 & -7 \\ -1 & 4 & 9 \end{pmatrix}$, find A^{-1}, the inverse matrix.

(10 marks)

Q4.

Given the vectors: A = (1, 3, -2) and B = (-1, 4, 2), Calculate:

(i) The Dot Product, A.B

(ii) The Cross Product, A x B

(iii) The angle between the two vectors, A and B.

(10 marks)

Q5.

(i) Create a modular 5 multiplication table.

(ii) What are the multiplicative inverses for modular 5?

(iii) Evaluate (23 + 8) mod 6.

(10 marks)

Class Test 3

Q1. Simplify the following expressions:

a) $\log_5(x+2) + \log_5(2x-1) - \log_5(x)$ using the rules of logarithms.

b) $\left(\frac{25}{64}\right)^{\frac{3}{2}}$ using rules of indices.

(10 marks)

Q2.

a) Solve for x: $2x^2 + 5x - 6 = 0$. Give your answer to 2 decimal places.

b) Solve for x: $3x^3 + 7x^2 - 7x - 3 = 0$.

(10 marks)

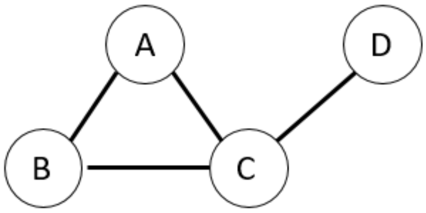
Q3.

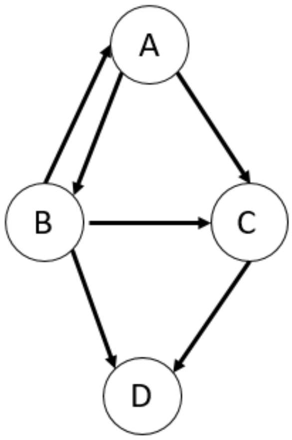
a) Draw the graph G = ({P, Q, R, S, U}, {{P, Q}, {P, R}, {Q, R}, {S, U}}

(5 marks)

b) Draw the adjacency matrix for each of the graphs below.

(2.5 marks each)





Q4.

a) What is a bipartite graph?

(5 marks)

b) Describe a method for determining if a graph is bipartite.

(5 marks)

Q5.

a) What is a planar graph?

(5 marks)

b) A planar graph has 5 vertices and 3 faces. How many edges does it have? Draw an example of this type of graph.

(5 marks)

GRIFFITH COLLEGE DUBLIN

QUALITY AND QUALIFICATIONS IRELAND
EXAMINATION

LINEAR ALGEBRA

Lecturer:

External Examiner(s):

Date: **XXXXXXXX** **Time: XXXXXXX**

THIS PAPER CONSISTS OF FIVE QUESTIONS
FOUR QUESTIONS TO BE ATTEMPTED
ALL QUESTIONS CARRY EQUAL MARKS
THE USE OF NON PROGRAMMABLE CALCULATORS IS PERMITTED
GRAPH PAPER TO BE SUPPLIED

***Note**:* Solutions will get credit for "correct method of working" and, where appropriate, for "checking the answer".

QUESTION 1

(a) Given the matrices **A, B** and **C** below:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -3 & 2 \\ 3 & 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -3 \\ 4 & 2 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$$

(i) Evaluate 6**A,** (if it is not possible give a brief explanation)

(2 marks)

(ii) Evaluate **B * A,** (if it is not possible give a brief explanation)

(2 marks)

(iii) Evaluate **C * A,** (if it is not possible give a brief explanation)

(3 marks)

(iv) Evaluate **A * B,** (if it is not possible give a brief explanation)

(3 marks)

(b) Given the following system of equations:

3x – 2y + 5z = 3

-2x + y + 4z = -2

x + 4y – 7z = 1

Using matrices and **Cramer's Rule** solve for x, y and z.

(15 marks)

Total (25 marks)

QUESTION 2

(a) Consider a triangle ABC with one angle, BAD = 45^o.

A perpendicular is dropped from B and meets AC at D.

The area of triangle BCD is 1.5 times the area of triangle ABD.

See diagram below.

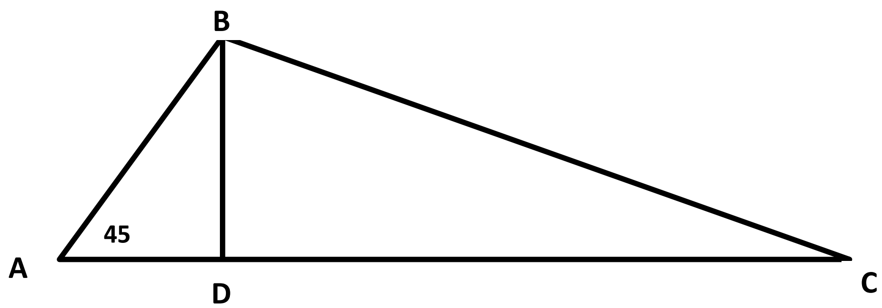


Diagram is **NOT** accurately drawn.

(i) Calculate the size of **angle ABC**, to two decimal places.

(5 marks)

(ii) Calculate the size of **angle ACB**, to two decimal places.

(5 marks)

(b) Given two vectors **a** = {3, 4, 0} and **b** = {4, 4, 2}.

(i) **Add** vectors **a** and **b**.

(2 marks)

(ii) **Subtract** vector **a** from vector **b**.

(3 marks)

(iii) Calculate the **cross product** of **a** and **b, a**x**b**.

(5 marks)

(iv) Find the **angle between the vectors a** and **b**, to two decimal places.

(5 marks)

Total (25 marks)

QUESTION 3

(a) Solve the following quadratic equation for x

$5x^2 + 6x = 8$

(5 marks)

(b) You are required to plot the graph of a function of x.

(i) Construct a table of values for x and y where $\mathbf{y = x^3 - 2x^2 - x + 2}$ for $\mathbf{-2 \leq x \leq +3}$, with **intervals of 0.5**.

You **must** show the intermediate values for each part of the function.

***Note**: you are advised to use 1 decimal places of accuracy for calculations.*

(12 marks)

(ii) Plot the graph of $\mathbf{y = x^3 - 2x^2 - x + 2}$ for $\mathbf{-2 \leq x \leq +3}$, with **intervals of 0.5**.

(8 marks)

Total (25 marks)

QUESTION 4

(c) Knowing that $\cos^2\theta + \sin^2\theta = 1$, prove each of the following identities:

(i) $\frac{1}{\tan(x)} + \tan x = \frac{1}{\sin(x)\cos(x)}$

(4 marks)

(ii) $\sin x - \sin x \cos^2 x = \sin^3 x$

(4 marks)

(d) Solve for x, where $\mathbf{\log_2(x) + \log_2(x-2) = 3}$

(6 marks)

(e) Evaluate the modulo arithmetic expressions, $\mathbf{121^3 \pmod{11}}$

(2 marks)

(f) Create a **modulo 8** multiplication table.

(5 marks)

(g) Find the multiplicative inverses for **modulo 7**.

(4 marks)

Total (25 marks)

QUESTION 5

(a) Where matrix **M** = $\begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

(i) Find the **inverse** of matrix **M** using Minors, Cofactors and the Adjugate (adjunct).

(16 marks)

(ii) Check your answer from (i), above

(3 marks)

(b) Consider the following matrices, **A** and **B**

A = $\begin{bmatrix} \mathbf{3} & \mathbf{4} & \mathbf{0} \\ \mathbf{5} & \mathbf{4} & \mathbf{2} \end{bmatrix}$

B = $\begin{bmatrix} \mathbf{-3} & \mathbf{4} \\ \mathbf{-5} & \mathbf{3} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix}$

(i) Calculate **A.B**, if it is not possible give an explanation why not.

(3 marks)

(ii) Calculate **B.A**, if it is not possible give an explanation why not

(3 marks)